On some parabolic properties of the bi-Laplacian operator

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Abstract

Recently the biharmonic operator has widely been studied in literature. It is worth noting that the obtained results have twofold for dimension bigger or smaller than the order of the operator. In our works we consider some extension of the existing theory in the two cases. In particular, we study the biharmonic operator perturbed by an inverse fourth-order potential in $\mathbb{R}^N$ with $N \geq 5$. More precisely, we consider $A = \Delta^2 - V = \Delta^2 - \frac{c}{|x|^4}$ where c is any constant such that $c < C^*$, $C^* = \left(\frac{N(N-4)}{4}\right)^2$. The semigroup generated by $-A$ in $L^2(\mathbb{R}^N)$, $N \geq 5$, extrapolates to a bounded holomorphic C_0-semigroup on $L^p(\mathbb{R}^N)$ for $p \in [p_0', p_0]$ where $p_0 = \frac{2N}{N-4}$ and p_0' is its dual exponent. Furthermore, we study the boundedness of the Riesz transform $\Delta A^{-1/2}$ on $L^p(\mathbb{R}^N)$ for all $p \in (p_0', 2]$.

We also consider an extension of the 1-dimensional theory studying the differential operator $A = \frac{d^4}{dx^4}$ acting on a connected network $\mathcal{G}$. We discuss self-adjointness issues, well-posedness of the associated linear parabolic problem and extrapolation of the generated semigroup to consistent families of semigroups on $L^p(\mathcal{G})$ for $1 \leq p \leq \infty$. Our most surprising finding is that, upon allowing the system enough time to reach diffusive regime, the parabolic equation driven by $-A$ may display Markovian features: analogous results seem to be unknown even in the classical case of domains.